Distribution network reconfiguration problem for energy loss minimization with variable load

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Abstract*— This paper presents a study on different method of modeling the network reconfiguration problem to take into account load variation during a period of time. This work aim to determine if optimization process needs to take into account this fact in order to be used by a real distribution network operator. It presents a way to optimize an integer objective function.The optimization problem of type Mixed Integer Quadratic Constrained Program (MIQCP) and Mixed Integer Non-Linear Program (MINLP) is presented. Three different process of optimization on a define period are presented, two avoiding the use of power flow calculation and one using it. Methods are computed using ***three different load scenarios on the famous Baran & Wu test case. And results tends to show that it is not useful to take into account load variation and worst case optimization is the best method.**

I. Introduction

This paper present a joint work between the DSO (Distribution System Operator) of the french department Vienne called SRD, and LIAS research lab. The aim of this document is to present results for loss minimization on a period of time in regards to future implementation in the real network.

Several method for loss reduction have already been proposed such as capacitor placement, distributed generation allocation or feeder reconfiguration [1].

Feeder reconfiguration by switch state modification have been well studied and different optimization methods have been used to solved this problem [1].

The majority of the work focused on instaneous power only, and neglected the variability of load demand as stated in [2]. In fact the power demanded is not the same at each moment of the day. Also focusing on the energy instead of power is more representative of the reality of DSOs. Because calculating configuration based on power means that this proposed configuration is valid for only the power taken into account and maybe is to conservative or it can not handle larger consumption.

But it is not possible to reconfigure the network oftenly during a short period of time due to two reason. Firstly remotely controlled switches will be damage to quickly and secondly it is not imaginable do send an agent to modify the manual switches several time during a short period. The first reason can be view as only an investment problem and maybe the gain in term of losses can compensate the switches renewing. And the second reason have already been studied [2] to determine which switch has to be modified in a network to become remotely controlled. And the switch modified can become inappropriate if the network evolve during his life. Also automatic switching for network reconfiguration have been studied in [3]. automatically changing network configuration is practically infeasible because security device has to be modified to handle the new load of the feeders so it could not be done so oftenly.

In our study we only focused on finding fixed configuration for certain period of time. In other words finding a way to optimize an integer objective function. And this model aimed to be used on real network for long-term (season) or for short-term (day, week) period. Also the proposed method have the objective to be use one large scale networks, so it has to be developed in order to avoid computation problems.

The optimization model is define in section II. The way of taking into account the energy instead of the power and detail about the variable load demand are presented in section III. Section IV present the results computed over Baran & Wu test case [4] with different scenarios.

II. Optimization model

The model presented bellow aimed to be used in an industrial context with a large network containing a large number of nodes and edges. In order to handle this size of problem some simplification are made. This is also due to the lack of information available in real distribution network. As reactive consumption and phase (φ) are unknown at each node of the network. Also u is considered constant. In fact in this study the model optimize a power flow only based on real power. Furthermore the energy losses on the line to be minimized can be deduced with the active power alone.

A. Power optimization problem definition

1) Glossary:

$x(i,j)$: Real value representing the power flow from vertex i to j

$ch(i)$: Real value representing relative voltage drop at node I
$b(i,j)$: Binary value representing line state (1=edges (i,j) active, 0= edge (i,j) not active)
$kp(i,j)$: Constant loss coefficient from i to j
$kc(i,j)$: Constant voltage drop coefficient from i to j
$up(i,j)$: Upper bound of x(i,j)
$up_{ch}(i)$: Upper bound of ch(i)
nb_{source} : Number sources
$src(p)$: Maximum power that a source p can deliver

a) objective function:

$$Z = \sum kp(i,j) * x(i,j) * b(i,j) \qquad \forall (i,j) \tag{1}$$

b) Constraint:

$$\sum_{i=0}^{n} x(i,j) \times b(i,j) - \sum_{k=0}^{n} x(j,k) \times b(j,k) = P(j) \tag{2}$$

$\forall j$ with $\forall k$ consecutive to j and $\forall i$ incident to j

$$\sum b(i,j) \leq 1 \ \forall j \text{ incident to } i \tag{3}$$

$$ch(i) = \sum (i,j) * (x(i,j) \times kc(i,j)) + ch(j)) \tag{4}$$

$\forall\ j$ incident to i

$$b(i,j) + b(j,i) \leq 1 \ \forall (i,j),\ \forall (j,i) \tag{5}$$

$$x(i,j) \leq up(i,j) \ \forall (i,j) \tag{6}$$

$$x(i,j) \geq -up(i,j) \ \forall (i,j) \tag{7}$$

$$ch(i) \leq up_{ch}(i) \ \forall i \tag{8}$$

$$ch(i) \geq -up_{ch}(i) \forall i \tag{9}$$

$$\sum b(p,i) = nb_{source} \ \forall (p,i) \ i \text{ consecutive to } p \tag{10}$$

$$x(i,p) \leq src(p) \ \forall (i,p) \ i \text{ incident to source } p \tag{11}$$

B. calculation of energy loss

In this section, we show how to compute the energy loss between two nodes i and j of the power network. And this calculation defines the objective function to minimize.

$$P_{loss}(i,j) = R(i,j) \times i(i,j)^2$$

$$i(i,j) = \frac{P(i,j)}{u \times \cos\varphi}$$

$$\text{so} \quad P_{loss}(i,j) = kp(i,j) \times P(i,j)^2 \tag{12}$$

$$\text{with} \quad kp(i,j) = \frac{R(i,j)}{u^2 \times (\cos\varphi)^2)} \tag{13}$$

where $R(i,j)$ is the resistance of the line connecting node i to j and $P(i,j)$ the power flowing between i and j.
The aim of this model is to be used to minimize the total amount of loss of the network caused by the loss of each individual line. normally it should be added to the real network flow, because in reality power loss indirectly increase the value of intensity in the line but in our study this fact is not taken into account.

$$P_{loss\_total} = \sum \quad P_{loss}(i,j) \quad \forall\ (i,j) \tag{14}$$

The distribution network is modeled by a graph composed by two type of nodes:

- Source nodes the High Voltage to Medium Voltage (HV/MV) transformers
- consumption nodes the Medium Voltage to Low Voltage (MV/LV) transformers or big consumer directly plugged onto the MV network

All those nodes are connected by edges having a fixed cost, the $kp(i,j)$ value (see Eq. 13). Each electrical line is represented by two edges to represent the two possible directions of the electrical flow. Each connected component composed of one source and all the consumption which get power from this source are called feeder in reality.
Note that a consumption node may also be a renewable energy source and in this case its consumption has a negative value. We can not count renewable sources as normal source because the power delivered by this source can be add to the power for the source. And we set in the model constraint that avoid powering on consumption by two different sources.

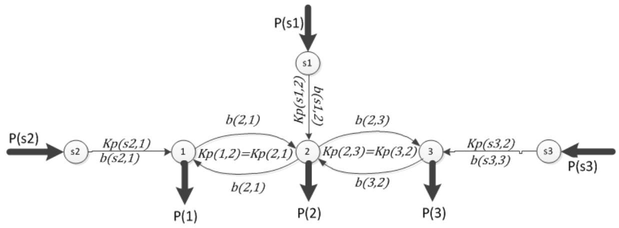

Fig. 1. Simplified network graph

Figure 1 show the graph representation of a simple distribution network. We assume that their is only one possible direction for the flow at the HV/MV transformers. But when in a feeder or all the feeders connected to this transformers the mean of the power produced is greater than the mean of the power consumed during the minimum period of measured time (10min). It is considered as a negative flow due to the fact that productions are computed as negative consumption.

C. Electrical flow computation

In section II-B the objective function is described and in the following sections the way to compute the power flowing through each line is presented. Also we explain how take into account the operability problem of the uniqueness of a source for a consumption.
Graph theory says that the flow coming into a node is equal to the flow going out, this is a rule applicable to active power flow or reactive power flow but not apparent power (Kirschofs law) [5]. Also the graph has to be conservative, so the graph

of Fig. 1 is defined as follows:

$$\begin{aligned}
\text{Node 1: } & P(s2,1) + P(2,1) = P(1,2) + P1 \\
& P(s2,1) + P(2,1) - P(1,2) = P1 \qquad (15) \\
\text{Node 2: } & P(1,2) + P(s1,2) + P(3,2) = \\
& P(2,3) + P(2,1) + P2 \\
& P(1,2) + P(s1,2) + P(3,2) \\
& -P(2,3) - P(2,1) = P2 \qquad (16) \\
\text{Node 3: } & P(s3,3) + P(2,3) = P(3,2) + P3 \\
& P(s3,3) + P(2,3) - P(3,2) = P3 \qquad (17) \\
\text{Conservation : } & P(s1,2) + P(s2,1) + P(s3,3) = \\
& P1 + P2 + P3 \qquad (18)
\end{aligned}$$

With equations 15-18 we know the flow going through the line of the network, but we still have to constraint this flow to be realizable. That is the aim of the operational constraints

D. Operability constraint

As stated above because of the fact that distribution network are used in radial configuration we have to find a way to constrained the search of a configuration in that way. That means simply that a consumption node cannot be powered by two different sources. In order to do that we have to introduce into the model a binary variable $b(i,j)$ representing whether an edge connects (i,j) or not. That is the set of b values that gives a network configuration by defining where the network should be opened and closed.

Based on figure 1 we can define the following equations:

$$\text{Node 1: } b(s2,1) + b(2,1) = 1 \qquad (19)$$

$$\text{Node 2: } b(1,2) + b(s1,2) + b(3,2) = 1 \qquad (20)$$

$$\text{Node 3: } b(s3,3) + b(2,3) = 1 \qquad (21)$$

Also another important rule has to be implemented, concerning the fact that two opposite edges cannot be active at the same time, which means that the power can only go in one direction through a line.

In the example figure 1 this applies to only two pairs of edges:

$$b(1,2) + b(2,1) \leq 1 \qquad (22)$$

$$b(2,3) + b(3,2) \leq 1 \qquad (23)$$

When the model is used for a real application, equations (22) and (23) are slightly modified. In fact in reality not all lines can be opened, because their isn't a switch at each side of each line. So the condition for allowing a line to be opened is that a switch belongs to the edge considered. If this condition is respected, the constraints are written as in 22 and 23. Whereas in the other case the inequality becomes an equality meaning that at least one of the two edges have to be connected.

Note that for computation problems $b(s1,2), b(s2,1)$ and $b(s3,3)$ are forced to one.

With these constraints described in above sections and knowing equation (12) the optimization model defined in section II-A can be defined with the variables $x(i,j) = P(i,j)$. Two different equations for energy loss calculation are used to simplify the computation, one taking into account the square of $x(i,j)$ and the other not.

By taking into account the square of the power in the objective function, the Mixed Integer Quadratic Constrained Problem (MIQCP) becomes a Mixed Integer Non-Linear Problem (MINLP), and due to the non-convexity it is harder to solve. We will see in the next sections how it affect the result of the optimization.

III. Optimization period modeling

The model described in the above sections is presented for power optimization, but in reality we must think in terms of energy. In general every cost associated with the electricity market is set in energy (kW/h), and power doesn't represent the total amount of energy transited and lost into a network during a period of time. So in order to realize the optimization on a defined period of time (day, week, season...), we have to find a way to model the integral of the power loss (Equation 12 presented in previous section II-A.

In real networks load curves are known only for some

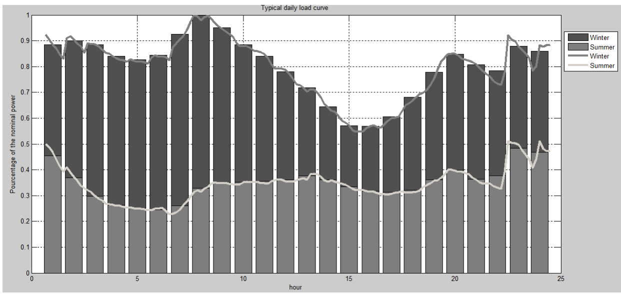

Fig. 2. typical daily load curve

consumptions like big industries. Global consumption curve can also be obtain at the high to medium voltage (HV/MV) transformers as the one presented on Figure 2. This figure is taken from load curve of one of DSO HV/MV transformer. Usually load curves are obtain by measuring power every ten minutes (light grey continuous curve), and the mean of this values is calculate using 6 points to defined an energy in kWh (dark grey bar graph). This particular value (dark grey bar) are called unitary-period value of the load curve in our study and are consistent with the usual energy unit (kWh). Their is a usual pattern in the daily load curve that can be used so we defined typical load curve of consumption for summer and winter days.

This curve representing the power consumption during 24 hours has to be integrate but we do not have the exact function of this curve. Therefore integration by part is used. Three methods are presented to handle that kind of integration into the optimization problem presented in previous section:

(a) Integrate all parts directly in the optimization problem, and deduce a configuration from this solution
(b) Solve an optimization problem for each unitary-period of the load curve and externally calculate the loss for this given solution over all the period

(c) solve the optimization problem for only one characteristic value of the load curve (min, mean, max)

A. Methods description

In this subsection we explain how the previous model of power loss optimization described in section II-A is modified to optimize energy loss for each methods (a), (b) and (c).

1) Method (a):

This seems to be the best way of modeling the integer behaviour of optimization in term of energy instead of power, because it calculate an configuration taking into account all unitary-period of the load curve. Indeed the objective function will sum all losses generated by each unitary-period into every line. So the result of the objective function should be equal to the energy losses.

- objective function (Eq. 1) modification for a h hours period

$$Z = \sum \Sigma_{p=0}^{n} kp(i,j) \times x_p(i,j) \times b(i,j) \qquad \forall (i,j) \quad (24)$$

  $x_p(i,j)$ represent the energy in the line (i,j) for the period k. But the fact is that every $x_p(i,j)$ have to be calculated
- Kirschofs law (Eq. 2) modification for h hours period

$$\sum_{i=0}^{n} x_p(i,j) \times b(i,j) - \sum_{k=0}^{n} x_p(j,k) \times b(j,k) = P_p(j) \quad (25)$$
$$\forall j \text{ with } \forall k \text{ consecutive to } j \text{ and } \forall i \text{ incident to } j$$

  This set of equations is defined for every p to compute every $x_p(i,j)$.

So with this method the optimization model is defined with $m \times h$ variables and $n \times h$ Kirschofs equations (m number of edges, n number of node).

2) Method (b):

In this method the model described in section II-A is unchanged. And one solution of the optimization problem is found for each unitary-period of the load curve. But it needs the use of an external power flow software. The only thing that change for every search is the value $P(j)$ in Equation (2) that is replaced by the unitary-period value. Also for each obtained solution a loss calculation is computed using an state estimator software (MATPOWER [6]) to get the energy losses generates by this configuration for the whole period. And when we found as much configuration as unitary-period we can deduce the configuration with the lesser losses in term of energy. This process is described in Figure 3.

Power flow calculation could be avoided by adding the same set of equations (2) with replacing $x(i,j)$ by another variable $y(i,j)$ and the maximum of the $p(j)$ value. It will give us the flow for the maximum consumption for the given configuration. And we can just compute in an another variable $Z^{'}$ the sum of the loss. This metric $Z^{'}$ will be used to compare directly each solution without the use of a powerflow tool. But it will add a lot of variable and computation time in a real network.

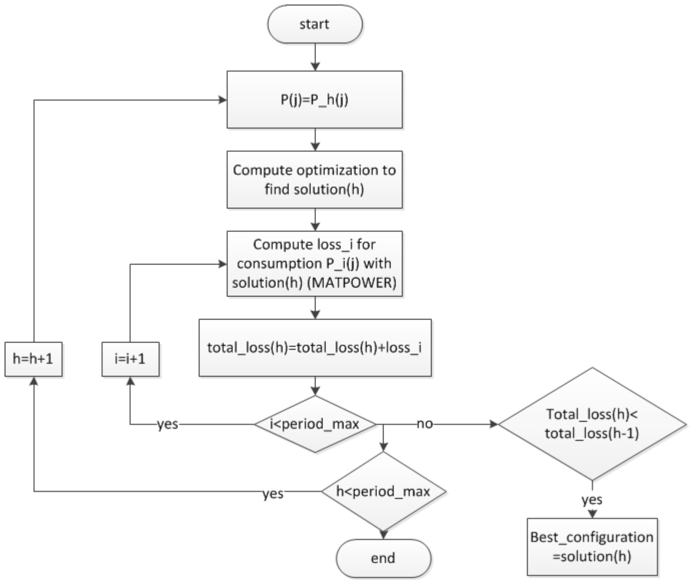

Fig. 3. method(b) process

3) Method (c):

The aim of this method is to check if the fact to take only an extreme value or a particular one (the mean) of the load curve is relevant to define a configuration that minimize the energy loss. As in the method (b) only $P(j)$ in the Equation (2) is modified by the min, max or mean of the load curve. And for each of these configuration the energy loss of the entire period are computed using MATPOWER.

IV. Methods computation results

The proposed model has been tested over Baran & Wu test case [4] composed of 33 nodes. In order to also test the impact of the load modeling on the solutions found we defined 4 different scenario test:

1) Two 24 hours load curve are used, one representing a typical summer day and on winter day. Each unitary-period is equal to the mean of 1 hour of load.
2) A total year load curve is dived in 2 seasons (winter and summer), each unitary-period is equal to the mean of load of 1 week.

To use this load curve as entry data for our optimization model we normalize the two load curve (winter and summer) by dividing the power by the maximum value of both summer and winter load curve. And this coefficient is applied to the nominal power of each consumption of the Baran & Wu test case.

In those two previous test case all consumption in the network have the same load profile. We assume that they consume the same amount of load proportionally to their nominal one. But we know that in reality every consumer of different kind (household, industry, farmers) have different load curve during the same day. So to try to model this behaviour we attribute

to each consumption a particular load curve, in accordance to their nominal consumption value.
The profile used in the study are taken from DSO data base. The wave form of these curves can be discuss but are used to test how the optimization algorithm react with it.

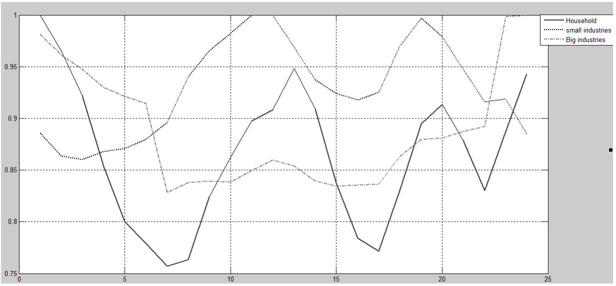

Fig. 4. profile

3) Three different load profile are used on a typical 24 hour winter day.

Also a fourth scenario have been tested in presence of renewable energy source.

4) All load have the same consumption, and one node becomes a solar production.

It has to be noted that all those four scenarios have been compared to the actual best configuration found in the literature for the instantaneous power optimization of baran & wu test case. But in the fourth scenario, this comparison has no real meaning because no production insertion have been made in the literature***a verifier***. The optimization is modeled through GAMS and is computed on NEOS server [7] [8].

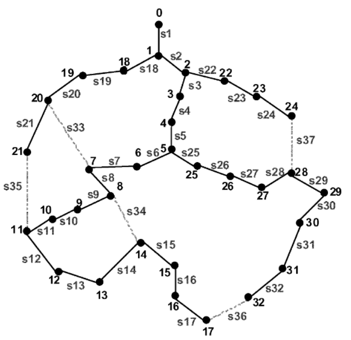

Fig. 5. Baran and Wu network

A. Scenario 1) and 2) results

Results of these two scenario are found in Table I and II. It should be noted that in case where simplification of the square on the power to compute losses is made, all three methods give the exact same solution.

With typical summer weekly and daily load curve the two scenarios shows that method (b) gives the best results in term of power losses. This method give this results twice during the search and can not be related to any specific value of the load curve. Also method (c) gives a result not so far from this previous value and correspond to the optimum value given by the literature. In fact the specific value that give the best result in method (c) is the maximum of the load curve so it corresponds to the basic test case. Whereas method (a) gives the worse solution.

In the case where typical summer daily and weekly load curve are taken the same observation can be made as with winter load curve. Whereas with method (c) the configuration found with the max value is better and don't correspond to the optimum configuration of the literature.

	Winter			
	Daily (71.7 MWh)		**Weekly (10963 MWh)**	
Method	**Loss (MWh)**	**Voltage (pu)**	**Loss (MWh)**	**Voltage (pu)**
Optimum configuration	2.573	0.9378	395.77	0.9378
Square simplification	2.836	0.9276	419.22	0.9354
(a)	2.7962	0.9291	419.22	0.9354
(b)	2.5647	0.9413	394.93	0.941
(c)	2.5733	0.9378	395.77	0.9378

TABLE I
METHODS RESULTS COMPARISON FOR WINTER VALUE

	Summer			
	Daily (31.1 MWh)		**Weekly (9084 MWh)**	
Method	**Loss (MWh)**	**Voltage (pu)**	**Loss (MWh)**	**Voltage (pu)**
optimum configuration	1.3673	0.958	344.43	0.9544
square simplification	1.477	0.9526	374.27	0.948
(a)	1.411	0.9553	374.27	0.948
(b)	1.3196	0.9526	336.17	0.9465
(c)	1.331	0.9608	344.43	0.9544

TABLE II
METHODS RESULTS COMPARISON FOR SUMMER VALUE

Also it should be noted that with method (a) in winter and summer in scenario (2) the best value is given with simplification on the square of the power for loss calculation. But it don't affect the comparison between the three methods.

In those two scenario we assume that all load of the network have the same load curve. In other word they consume the same amount proportionally to their nominal value. But we know that in reality it never occur so in next subsection we assign load profile to each node. Also we will focus only on daily load curve because the period chosen don't affect the comparison between the three methods.

B. Scenario 3) results

In this third scenario the behaviour is slightly different from previous scenario 1) and 2). In fact configuration given by method (b),(c) and the optimum configuration are the same. For method (c) this behaviour is normal because it is the max value that give this configuration which is the same as the case used in the literature.

	Winter	
	Daily (78.6 MWh)	
Method	**Loss (MWh)**	**Voltage (pu)**
optimum configuration	2.8503	0.9390
square simplification	3.2723	0.9244
(a)	3.2723	0.9244
(b)	2.8503	0.9390
(c)	2.8503	0.9390

TABLE III
METHODS RESULTS COMPARISON FOR WINTER VALUE WITH PROFILE

C. Comparison of three methods

The results presented above tend to show that the best way to take into account the variability of the load is to find one configuration for each level of the load curve and choose the one that minimize the losses on the rest of the period (method (b)). But this approach is good only in a small test case like the one presented here with small computation time. Because if you want to optimize a network for one year or for a season and still use hourly dived load curve computation time can become a limitation. In fact it create as many optimization problem as unitary-period of the load curve.

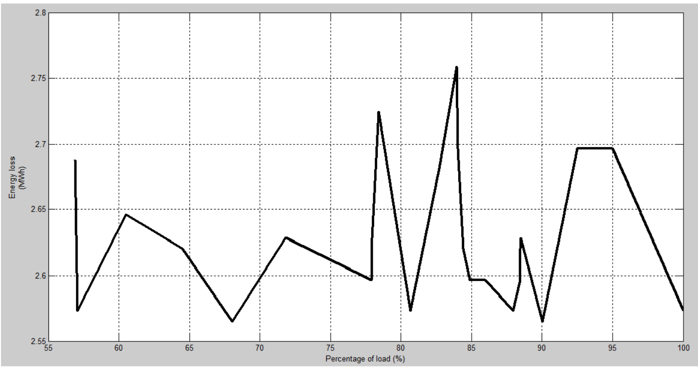

Fig. 6. Losses in function of load (winter day case)

Also it should be noted that it need the use of an additional tool to compute the loss. And in an industrial context this sort of tools are not always available.

Also we would think that method (b) could be used to define the best level of load curve to use to find the best solution. But Figure 6 shows no linearity or relation between load and losses.

Method (c) has more advantages because it's easiest to implement. It doesn't need any load curve only the maximum value of the load curve can be used to define the best configuration for the period.

It is also easy to compute in term of time taken by the optimization solver. And by taking the maximum of the load curve you are sure to be able to handle the worst case of consumption.

Method (a) seems to be to complicated to be used because it hasn't good performance in term of losses optimization and computation time. Whereas it's the only method that test in one optimization procedure all the load constraint of the period.

V. CONCLUSION

This study proofs that in a specific industrial context it is more suitable to simplify the load modeling in the optimization model. It is also shown that it is not necessary to model the integral on the power loss calculation to have good results on energy loss reduction. This study used a specific optimization model and specific profile and load curves that could be different for another kind of network (more urban than rural). In fact even if best results are found with method (b) it is the hardest to implement in term of scaling to large scale problem. Also taking into account individual profile for each node consumption has not show particular importance. However these observations are valid for small network but may be different for large scale networks. But other works have shown the same results [9].

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